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The effects of a two-dimensional, cylindrically symmetric periodic structure on the radiation from a source located inside the structure are considered. The coupling of the source radiation to the cylindrical cavity is analyzed classically by consideration of the interaction of a current line source with its own radiated field supported by the cavity. The analysis predicts variations in the radiative damping rate (inverse lifetime) and associated shifts in the oscillation frequency of the source. It is found that significant enhancement and inhibition of the radiation are possible even when the source is not at the center of the structure. Frequency shifts induced by the cavity are found to be negligible relative to the size of the band gap of the Bragg structure. The class of periodic structures analyzed has potential application to planar waveguide devices, such as concentric-circle gratings, surface-emitting lasers, and cylindrically symmetric lasers and amplifiers.

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Enhancement and inhibition of radiation in cylindrically symmetric, periodic structures

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The effects of a two-dimensional, cylindrically symmetric periodic structure on the radiation from a source located inside the structure are considered. The coupling of the source radiation to the cylindrical cavity is analyzed classically by consideration of the interaction of a current line source with its own radiated field supported by the cavity. The analysis predicts variations in the radiative damping rate (inverse lifetime) and associated shifts in the oscillation frequency of the source. It is found that significant enhancement and inhibition of the radiation are possible even when the source is not at the center of the structure. Frequency shifts induced by the cavity are found to be negligible relative to the size of the band gap of the Bragg structure. The class of periodic structures analyzed has potential application to planar waveguide devices, such as concentric-circle gratings, surface-emitting lasers, and cylindrically symmetric lasers and amplifiers.

1. INTRODUCTION

It is well understood that the radiative decay of excited atoms through spontaneous emission can depend strongly on the electromagnetic modes associated with the atoms' surrounding environment.¹ The spontaneous emission might be either enhanced or inhibited, depending on the characteristics of the atom and the cavity. Many investigations of this phenomenon have been concerned with the effects of metal and dielectric interfaces, and metallic waveguides near cutoff, on atomic radiation.^{2,3} A beautiful series of experiments by Drexhage⁴ and others examined the dependence of the fluorescent lifetime of excited molecules on the separation from gold, silver, and copper interfaces. While they are fundamentally highly significant, metallic structures are less important in practice because of their inherently lossy nature. As a result periodic dielectric structures have received considerable attention recently. Most investigations of these structures have been concerned with one-dimensional and three-dimensional rectangular periodicities—the one-dimensional case has direct applications to vertical-cavity surface-emitting semiconductor lasers;^{5,6,7} the three-dimensional structure offers the possibility of a total photonic band gap, in which electromagnetic wave propagation is isotropically forbidden within an energy gap of nonzero width.⁸ Two-dimensional (2D) periodic structures have also received recent interest.^{9,10} In addition to their fundamental interest, these structures may have applications in planar waveguides and in lasers and amplifiers that operate along the nonperiodic axis (since spontaneous emission is inhibited in the periodic plane). For model systems of dimensions higher than one, previous research focused on calculating the energy gaps for various types of translationally invariant systems. Of equal interest, however, is the problem of an elementary source placed within a Bragg-reflecting structure, a complex problem for practical systems. In this paper we consider the possibility of altering radiative properties in two dimensions by using only a 1D periodicity, that is, along

the radial direction in a cylindrically symmetric, periodic structure.

Cylindrically symmetric, periodic structures have already received considerable attention because of their potential to form circularly symmetric, surface-emitting semiconductor laser resonators.^{11,12} These lasers employ a radial surface corrugation on a planar waveguide. An example of a typical structure fabricated for use in a circular-grating laser is shown in Fig. 1. These gratings were defined by electron-beam lithography and were subsequently etched into the AlGaAs laser material by chemically assisted (chlorine) ion-beam etching.¹³ Recently the first observation of true circular-mode lasing in one of these lasers was reported.¹⁴ For this case, in which lasing occurs along the radial direction, it is most desirable to enhance spontaneous emission, since it has been shown that one can decrease the threshold of a laser substantially by increasing the coupling efficiency of spontaneous emission into a lasing mode.⁶

Because of the fundamentally unique nature of the cylindrically symmetric periodicity and the opportunity for insight into practical applications such as the circular-grating laser, it is interesting to consider the cavity effects of such a structure on the radiation of an enclosed source. At first thought it would seem that the source would have to be placed at the center of the structure in order for it to see the periodicity and strongly couple to the cavity or to exhibit significant enhancement or inhibition of radiation. However, for a cylindrical periodic structure of infinite radial extent, any point at a finite radius appears to be at the center when viewed from infinity: the radiation from a source placed at a nonzero but finite radial position can be described by a finite number of azimuthal waves, all of which approach circular waves as they radiate out to infinity. Therefore a cylindrical periodic structure of large but finite radial extent ought to couple strongly to radiation emitted from a region of nonzero extent around the center. It is our intent to show that this region does exist and can extend over a distance of many wavelengths, indicating that practical

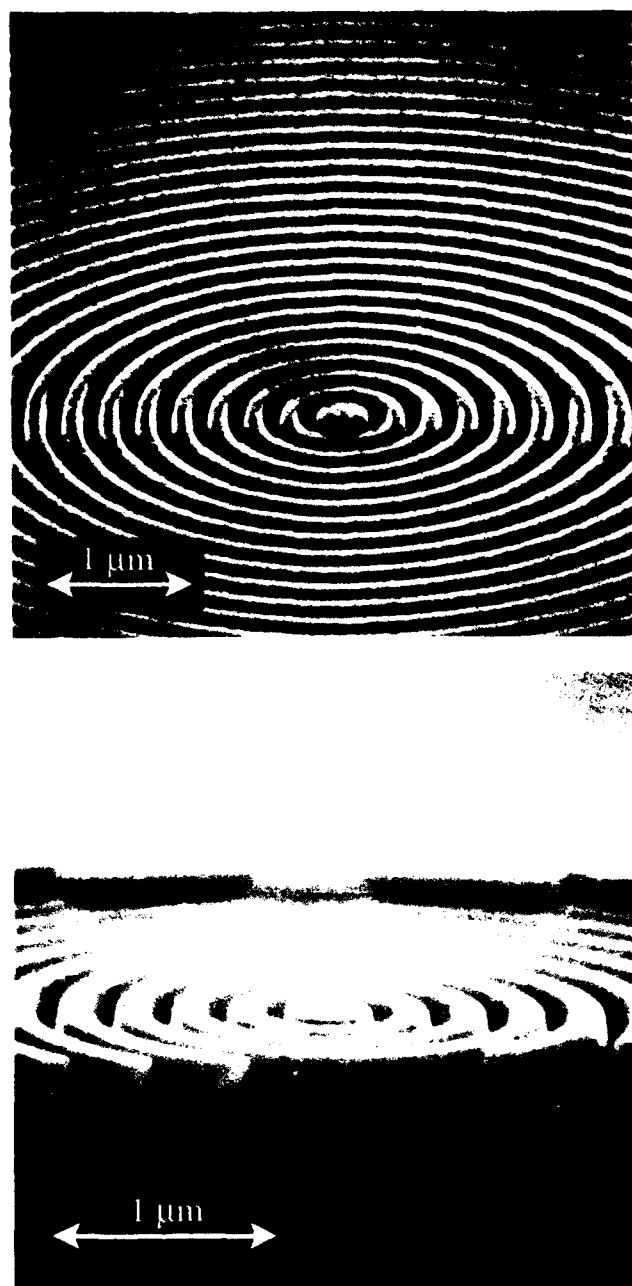


Fig. 1. Scanning electron micrographs of a typical grating resonator for a concentric-circle grating, surface-emitting semiconductor laser.

applications involving the cylindrical periodic structure are feasible.

Because the unique radiative effects associated with cylindrically symmetric, periodic structures are a direct result of the 2D circular nature of these structures, we examine a strictly 2D system in this paper; i.e., we assume a current line source that emits strictly cylindrical waves, so that both source and cavity are translationally invariant along the cylinder axis. This simplification ignores radiation that propagates along the cylinder axis. For a practical, realizable structure with cylindrical symmetry, accurate predictions of the magnitudes of the radiative effects should take into account the existence of radiative channels associated with propagation in the axial direc-

tion. Thus, while the theory in this paper is not capable of quantitatively predicting radiative effects in devices like that shown in Fig. 1, it is likely that the qualitative features emerging from the model will offer insight into the behavior of such devices. This is particularly true for cylindrical structures that include a planar optical waveguide, as a result of the significant coupling between the radially propagating bound modes and a source located on or within the waveguide.¹⁵ In this context the theory in this paper can be viewed as a first step toward understanding the radiative properties of practical structures, such as those involving concentric-circle gratings, by examining a simpler system for which essentially exact results can be obtained.

The effect of a cavity on the enhancement or the inhibition of spontaneous emission can be theoretically modeled in many ways. A simple and intuitive model treats the source as a classical dipole oscillator that is driven by the reflection of its own radiated field from the cavity. This method was employed by Kuhn to describe the effect of a metallic mirror on the radiation from a nearby excited molecule.¹⁶ It was later used quite successfully by Chance *et al.*¹⁷ to model quantitatively the experimental data of Drexhage. We adapt this classical method to analyze the effects of the cylindrical periodic structure on the radiation of an enclosed source for several reasons. First, the method allows us directly to analyze radiative effects as a function of source position in the cavity and hence to address questions regarding the size of the central region for which significant effects can occur. Second, it is not clear how a band-theory description of electromagnetic wave propagation can be applied to the cylindrical case, as is often done in the case of the rectangular periodicity. In particular, many concepts in band theory, such as reciprocal space and the Brillouin zone, rely on the translational invariance inherent in rectangular periodic systems. In the cylindrical system, if one moves an integer number of periods along the radius, the surrounding structure changes, and hence the system is not translationally invariant.

The organization of this paper is as follows. In Section 2 we develop the theoretical formulation for calculation of the enhancement and inhibition of cylindrical-wave radiation from a current line source in terms of the radiative damping rate (inverse lifetime) and the associated frequency shifts. The formulation uses cylindrical coupled-wave theory to calculate the reflection of the radiated field back onto the source.^{11,18} In Section 3 the theory is used to calculate numerically radiation enhancement and inhibition and frequency shifts for representative structures. Finally, in Section 4 conclusions are drawn from the results of this investigation.

2. THEORETICAL FORMULATION

In this analysis we treat the source of radiation as a classical current line source. We model the effects of the cylindrical periodic structure on the source, following Kuhn's method.¹⁶ In Subsection 2.A we describe the equation of motion of the current line source, and from that equation we derive the changes in radiative damping rate (inverse lifetime) and frequency from the free-space quantities that are due to the source's own field reflected from the

cavity. In order to calculate the reflected field, we must first calculate the field emitted by the source; this field is derived in Subsection 2.B. In Subsection 2.C we derive the reflected field at the source position by using cylindrical coupled-wave theory. Finally, in Subsection 2.D, complete expressions for the radiative damping rate and the frequency shift are derived.

A. Source Equation of Motion

In Kuhn's analysis the source is assumed to be a harmonically bound charge that represents a point dipole. In the present 2D analysis the reduction in the order of the problem dimensionality must be accompanied by a redefinition of characteristic source parameters in terms of a unit length. We assume that the current line source behaves as an ensemble of harmonically bound charge elements, described by the charge per unit length, q , such that this ensemble obeys the equation of motion

$$\ddot{p} + \omega_0^2 p = (q^2/m)E_R - b_0 \dot{p}, \quad (1)$$

where p is the total dipole moment per unit length, ω_0 is the free-space oscillation frequency in the absence of all damping, m is the effective mass per unit length, E_R is the reflected field at the source position, and b_0 is the damping constant (inverse lifetime) in the absence of the cavity. The dipole moment per unit length, p , and the reflected field, E_R , oscillate at the same (complex) frequency:

$$p = p_0 \exp[-(i\omega + b/2)t], \quad (2a)$$

$$E_R = E_0 \exp[-(i\omega + b/2)t], \quad (2b)$$

where ω and b are the frequency and the damping rate in the presence of the cavity, respectively. The problem is then one of a driven harmonic oscillator, where the external force is proportional to the reflected radiation field of the line source.

Using the forms given by Eqs. (2) in Eq. (1) and equating real and imaginary parts independently, we arrive at the expressions

$$\frac{b}{b_0} = 1 + \frac{q^2}{m\omega p_0 b_0} \text{Im}(E_0), \quad (3)$$

$$\omega^2 - \omega_0^2 = \frac{b^2}{4} - \frac{bb_0}{2} - \frac{q^2}{mp_0} \text{Re}(E_0). \quad (4)$$

Equation (3) gives the normalized (to the free-space value) radiative damping rate. Equation (4), which gives the frequency shift caused by the presence of the cavity, can be simplified by recognizing that $\Delta\omega \ll \omega, \omega_0$, where $\Delta\omega \equiv \omega - \omega_0$. Using this assumption, which will be justified below, we can write Eq. (4) as

$$\Delta\omega \approx \frac{b^2}{8\omega_0} - \frac{bb_0}{4\omega_0} - \frac{q^2}{2m\omega_0 p_0} \text{Re}(E_0). \quad (5)$$

In order to calculate the radiative damping rate and frequency shift determined by Eqs. (3) and (5), respectively, we need to know the reflected field strength at the source, E_0 . It is convenient to factor this quantity, such that

$$E_0 = E_S \Gamma. \quad (6)$$

The quantity E_S , which denotes the source field strength, will be calculated in Subsection 2.B of this section. The

quantity Γ is an effective reflection coefficient that depends on the source position and the characteristics of the cylindrical periodic structure. In Subsection 2.C we show how this quantity can be computed with coupled-wave theory.

B. Source Field

Reducing the problem to one of strictly 2D character, we take the source to be a current line source oriented along the cylinder axis of the cylindrical periodic structure. The geometry of the problem is shown in Fig. 2. The source is described by the current density

$$\begin{aligned} \mathbf{J}(r, \phi) &= -i\omega_0 p_0 \delta(\mathbf{r} - \mathbf{r}_0) \hat{z} \\ &= -i\omega_0 p_0 (1/r) \delta(r - r_0) \delta(\phi - \phi_0) \hat{z}, \end{aligned} \quad (7)$$

where $\mathbf{r}_0 = (r_0, \phi_0)$ is the location of the source and $\delta(\mathbf{r})$ is the 2D Dirac delta function.

Because of the simple geometry of the source and the periodic structure, we need to consider only a single vector component of the electric field, $\mathbf{E} = \hat{z}E_z(r, \phi)$, which is a solution to the wave equation

$$\nabla \times \nabla \times \mathbf{E}(r, \phi) - \mu_0 \epsilon \omega^2 \mathbf{E}(r, \phi) = i\mu_0 \omega_0 \mathbf{J}(r, \phi), \quad (8)$$

where an $\exp(-i\omega_0 t)$ harmonic time dependence is assumed and ϵ is the constant permittivity in the source region ($r < r_1$). From the fact that the electric field has only a \hat{z} component, Eq. (8) is simplified to a 2D Helmholtz equation with a point-source driving term:

$$(\nabla^2 + k_0^2)E_z(r, \phi) = -\mu_0 \omega_0^2 p_0 \delta(\mathbf{r} - \mathbf{r}_0), \quad (9)$$

where $k_0 = (\mu_0 \epsilon)^{1/2} \omega_0$. The solution to such an equation is well known to be

$$E_z(r, \phi) = E_S H_0^{(1)}(k_0 |\mathbf{r} - \mathbf{r}_0|), \quad (10)$$

where

$$E_S \equiv \frac{ip_0 \omega_0^2}{4\epsilon_0 c^2} \quad (11)$$

and $H_0^{(1)}$ is a Hankel function of the first kind, order zero.

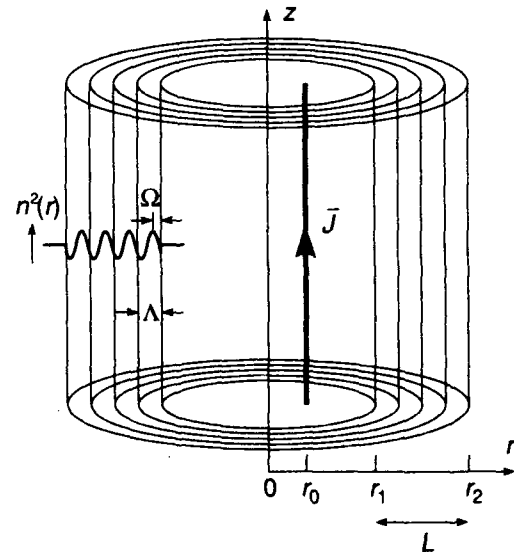


Fig. 2. Illustration of the geometry of the current line source and the cylindrical periodic structure. Both source and structure extend uniformly to $z = \pm\infty$.

Although Eq. (10) accurately describes the field emitted by the line source, it is not the most convenient form. By expanding the ϕ dependence of the field in a Fourier series, one can express the source field as

$$E_z^S(r, \phi) = E_S \sum_{m=-\infty}^{\infty} \exp[i m(\phi - \phi_0)] \times \begin{cases} J_m(k_0 r) H_m^{(1)}(k_0 r_0) & r < r_0, \\ J_m(k_0 r_0) H_m^{(1)}(k_0 r) & r > r_0, \end{cases} \quad (12)$$

where J_m is a Bessel function of order m . For $r > r_0$, the source field can be compactly written as

$$E_z^S(r, \phi) = E_S \sum_{m=-\infty}^{\infty} C_m H_m^{(1)}(k_0 r) \exp(i m \phi) \quad r > r_0, \quad (13)$$

with

$$C_m \equiv \exp(-i m \phi_0) J_m(k_0 r_0). \quad (14)$$

Equations (13) and (14) clearly demonstrate the nature of the source field emitted by the line source. For $r > r_0$, the field is composed of a sum of individual, outward-going azimuthal waves of order m , whose strengths are given by the amplitudes C_m . Even though the sum is infinite, for a finite source position r_0 the amplitudes C_m are appreciable only for a finite number of waves; therefore the infinite sum can be approximated well by a finite number of terms. Note that the source field is composed of standing waves for $r < r_0$ and hence is well behaved at the origin for a finite, but nonzero, source position.

C. Reflected Field

In this subsection we calculate the effective reflection coefficient Γ defined in Subsection 2.A. The basic idea is to express the source field as a sum of individual azimuthal wave components (as in Subsection 2.B), to calculate the strengths of those components at the inner radius of the cylindrical periodic structure ($r = r_1$), to compute the Bragg reflection of each azimuthal component separately by using cylindrical coupled-wave theory, and then to sum the reflected components back at the source to obtain the total reflected field.

In the source region, defined such that $r < r_1$, the total electric field E_z^T can be expressed as the sum of the source field E_z^S and a reflected field E_z^R , consisting of outward- and inward-going cylindrical waves generated by reflections off the cylindrical Bragg structure:

$$E_z^T = E_z^S + E_z^R, \quad (15)$$

with E_z^S given by Eq. (12) and

$$E_z^R = \sum_{m=-\infty}^{\infty} [a_m H_m^{(1)}(k_0 r) + b_m H_m^{(2)}(k_0 r)] \exp(i m \phi), \quad (16)$$

where a_m and b_m are (constant) outward- and inward-going wave amplitudes. The amplitudes a_m and b_m can be determined from the boundary conditions on the source region. At the center, $r = 0$, the sum of all outward-going waves of order m must be equal to the sum of all inward-going waves of order m . This intuitive boundary condition, indicating perfect reflection at the center, is mathematically necessary to prevent the total field from diverging at the center owing to the singular nature of cylindrical Neumann functions.¹⁸ Since this boundary condition is already built into the source term of the total

field, application to Eq. (16) requires that

$$a_m = b_m. \quad (17)$$

The other boundary condition is applied at the inner radius of the cylindrical Bragg structure, $r = r_1$. It requires that the sum of all inward-going waves of order m be equal to the sum of all outward-going waves of order m multiplied by the amplitude reflection coefficient, ρ_m , of the Bragg structure for a wave of order m :

$$b_m H_m^{(2)}(k_0 r_1) = \rho_m [a_m H_m^{(1)}(k_0 r_1) + E_S C_m H_m^{(1)}(k_0 r_1)]. \quad (18)$$

The reflection coefficient ρ_m is calculated by using scalar, cylindrical coupled-wave theory.¹⁹ This development uses a field expansion of the form of Eq. (16), where the outward- and inward-going wave amplitudes become functions of the radial coordinate to allow for the coupling induced by the periodic structure in the region $r_1 < r < r_2$. The coupled-wave equations for these outward- and inward-going field amplitudes in the structure, $A_m(r)$ and $B_m(r)$, respectively, are given by

$$\frac{dA_m}{dr} = i \frac{\pi k_0 r}{2} K(r) H_m^{(2)}(k_0 r) \times [A_m(r) H_m^{(1)}(k_0 r) + B_m(r) H_m^{(2)}(k_0 r)], \quad (19a)$$

$$\frac{dB_m}{dr} = -i \frac{\pi k_0 r}{2} K(r) H_m^{(1)}(k_0 r) \times [A_m(r) H_m^{(1)}(k_0 r) + B_m(r) H_m^{(2)}(k_0 r)]. \quad (19b)$$

These equations are solved numerically in the periodic region with the boundary conditions $A_m(r_1) = 1$ and $B_m(r_2) = 0$, since no inward-going waves are incident upon the structure from infinity. In these equations

$$K(r) = 2\kappa \cos\left(\frac{2\pi}{\Lambda} r - \Omega\right), \quad (20)$$

where Λ is the period of the Bragg structure, Ω is the phase of the periodicity with respect to the center, and the coupling constant κ is given by

$$\kappa = \frac{k_0}{4n^2} \Delta(n^2). \quad (21)$$

In this expression n is the bulk index of refraction of the source region ($r < r_1$) and $\Delta(n^2)$ is the index modulation in the periodic region ($r_1 < r < r_2$). The reflection coefficient is then given by

$$\rho_m = \frac{B_m(r_1)}{A_m(r_1)}. \quad (22)$$

Equations (17) and (18) can be solved simultaneously to yield the constant amplitudes a_m and b_m . These can then be inserted into Eq. (16), giving the reflected field everywhere in the source region. Evaluating this expression at the source position and recognizing that $E_0 = E_z^R(r_0, \phi_0) = \Gamma E_S$, we obtain

$$\Gamma = \sum_{m=-\infty}^{\infty} \frac{2\rho_m H_m^{(1)}(k_0 r_1)}{H_m^{(2)}(k_0 r_1) - \rho_m H_m^{(1)}(k_0 r_1)} [J_m(k_0 r_0)]^2 \quad (23)$$

for the effective reflection coefficient of the radiated field at the source position. For a given r_0 , the Bessel function J_m will eventually tend to zero for large enough m , so that the infinite sum in Eq. (23) can be well approximated by a finite sum.

D. Radiative Damping Rate and Frequency Shift

From Eq. (3) it is apparent that, to calculate the actual damping rate b , we need to know the free-space damping rate b_0 for the current line source. Because the line source in free space is radiating energy, its oscillation must be damped. One can calculate the rate of damping approximately by using a classical radiation reaction argument.²⁰ Starting with the electric field given in Eq. (10), the far-field Poynting vector for the line source in free space can be calculated and then integrated over the azimuthal angle to yield the power per unit length emitted by the source:

$$P = \frac{2}{\mu_0 \omega_0} |E_S|^2. \quad (24)$$

By application of the usual radiation-reaction argument, which is predicated on balancing the rate of energy lost to radiation with the rate of work done on the source by a reaction force, an approximate damping term is incorporated into the source equation of motion. This development yields an explicit expression for the free-space damping rate:

$$b_0 = \frac{q^2 \omega_0}{4\epsilon_0 mc^2}. \quad (25)$$

It is important to note that this damping rate is directly proportional to the frequency ω_0 , whereas the damping rate for a point dipole in free space is proportional to the square of the frequency. This difference is representative of a fundamental scaling because of the reduced dimensionality of the problem.

Substituting Eqs. (25), (11), and (6) into Eq. (3), we arrive at the simple expression for the radiative damping rate in the presence of the cavity

$$b/b_0 = 1 + \text{Re}(\Gamma), \quad (26)$$

where the approximation $\omega \approx \omega_0$ is used. Because $\text{Re}(\Gamma)$ spans the range $(-\infty, \infty)$, the normalized damping rate b/b_0 will span the physically meaningful range $(0, \infty)$. Equation (26) is the expression that we use in Section 3 to calculate radiative damping rates numerically. This expression clearly shows that the damping rate is proportional to reflected field strength. Therefore, when $b/b_0 \rightarrow 0$, or when there is a field node at the source position, the radiative lifetime becomes large, indicating inhibition of the radiation. As $b/b_0 \rightarrow \infty$, or when there is an antinode, the radiative lifetime becomes small, indicating enhancement.

The frequency shift can be calculated from Eq. (5) by the use of Eqs. (26), (25), (11), and (6), yielding

$$\Delta\omega \approx \frac{b_0^2}{8\omega_0} \{[\text{Re}(\Gamma)]^2 - 1\} + \frac{b_0}{2} \text{Im}(\Gamma). \quad (27)$$

Note that the classical frequency shift exactly agrees with the quantum-mechanically calculated level shift for the first excited state when the source is described as a harmonic oscillator.²¹ For a more realistic model of an atomic source, one needs a thorough quantum-mechanical calculation to predict the level shift accurately. To examine the magnitude of the frequency shift, consider a harmonically oscillating line source emitting at a free-space wavelength of $\lambda_0 = 0.5 \mu\text{m}$. In this case

$b_0 = 0.9 \times 10^8 \text{ Hz}$ and $\omega_0 = 0.4 \times 10^{16} \text{ Hz}$. Therefore $b_0 \ll \omega_0$, and relation (27) is well approximated by

$$\Delta\omega \approx (b_0/2) \text{Im}(\Gamma). \quad (28)$$

Thus, for most of the range of Γ , $\Delta\omega \sim b_0$, indicating that $\Delta\omega \ll \omega_0$, as we have assumed above. The possibility of large frequency shifts is addressed more carefully in Section 3 when we consider numerical results.

3. NUMERICAL RESULTS AND DISCUSSION

In this section the capability of a cylindrical Bragg structure to enhance or to inhibit source radiation is investigated numerically for some representative cases. We are concerned mostly with the radiative damping rate, b , as a function of various parameters, such as source position and the modulation, the placement, and the size of the Bragg structure. Most of the cases considered below involve structures with only 10 Bragg periods, which is perhaps the lower limit that one might employ in any practical structure, since significant reflectivity from a 10-period grating requires large modulation, or coupling constant κ . While this small number of periods was chosen mostly for convenience and for best illustration of the radiative effects, it is important to note that there is nothing in the theoretical formulation that limits the amount of modulation for which the theory is accurate. Even though we employ coupled-wave theory, the theory is not perturbative (as in the case of an actual waveguide grating), and we have made no slowly varying envelope approximation. Also, most of the cases below place the beginning of the Bragg structure at least 10 periods from the center. This choice allows us to examine a reasonably large range of source positions, since the developed theory inherently requires the source to be inside the inner radius of the Bragg region ($r_0 < r_1$). However, a more general treatment could be extended to include the possibility of source placement within the Bragg region.

We first examine the dependence of the radiative damping rate on the phase of the cosinusoidal periodicity. We assume that the source is exactly tuned to the Bragg frequency, or $\omega_0 n = \omega_B$, where $\omega_B \equiv \pi c/\Lambda$. The phase Ω is defined by Eq. (20) and is shown schematically in Fig. 2. This dependence is plotted in Fig. 3 for the source positioned exactly at the center ($r_0 = 0$). The Bragg region extends 10 periods along the radial direction and consists of three different modulation strengths, denoted by the coupling-constant-grating-length products κL of 1, 2, and 4. Over most of the full range of the grating phase Ω the source experiences inhibited radiation, since $b/b_0 < 1$; the stronger the modulation, the greater the degree of inhibition. Near $\Omega = \pi/2$, or for a sinusoidal grating, the source experiences strongly enhanced emission. Note that the enhancement resonance becomes significantly sharper for stronger modulations. Also shown in Fig. 3 is the case of a 100-period Bragg region with modulation strength $\kappa L = 2$. The behavior is similar to that of the 10-period case except for a slight shift in the enhancement resonance.

In Fig. 4 we examine the dependence of radiative damping rate on the source position. To examine the size of the source region over which significant enhancement can

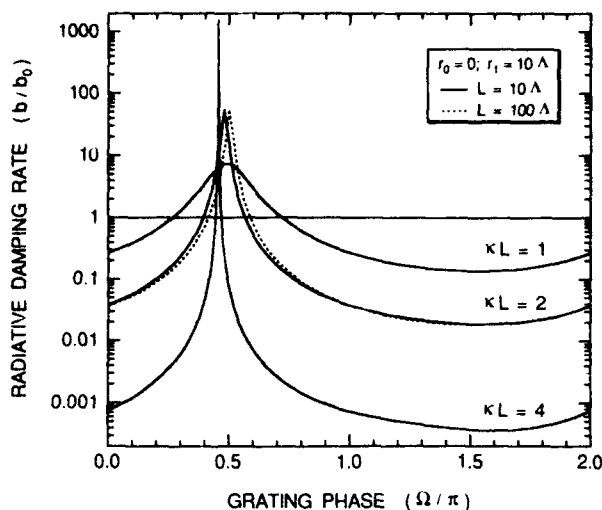


Fig. 3. Normalized radiative damping rate versus grating phase for a full range of phases ($0 \rightarrow 2\pi$) and source placement at the center.

occur, we fix the grating phase at the peak of the enhancement resonance in Fig. 3 (for $r_0 = 0$). This phase is denoted $\Omega = \Omega_{\max}$, where Ω_{\max} is a function of the modulation strength κL . Figure 4(a) shows the radiative damping rate for all possible source positions inside the inner radius of the Bragg structure ($r_0 < r_1$). It is apparent that, no matter where the source is placed, the source experiences significant enhancement for this particular choice of grating phase. The largest enhancement occurs at the center, since the source emits a perfectly cylindrically symmetric field from that position, and then the enhancement decreases as the source is moved away from the center. Although the strong modulation case ($\kappa L = 4$) experiences the greatest degree of enhancement, the enhancement oscillates dramatically with source position because of the sharpness of its resonance.

In Figs. 4(b) and 4(c) we examine the size of the source region over which significant inhibition can occur. In Fig. 4(b) we fix the grating phase at the point of minimum radiative damping (maximum inhibition) in Fig. 3. This phase is denoted $\Omega = \Omega_{\min}$. The radiative damping rate is plotted for all possible source positions inside the inner radius of the Bragg structure. The plot shows that the source experiences significant inhibition for most source positions. In fact there is a region with a radius of approximately two full grating periods for which the source experiences essentially the same inhibition as it does when placed exactly at the center. As in Fig. 4(a), stronger modulation produces a greater degree of inhibition and greater oscillation of inhibition with source position. This oscillation is not absolutely necessary, though. In Fig. 4(c) we fix the grating phase at $\Omega = 3\pi/2$, which is near the maximum inhibition phase but not exactly equal to it, and we perform the same calculation as that in Fig. 4(b). We find that the degree of inhibition is in general greater over the range of source positions than that for $\Omega = \Omega_{\min}$, and, more important, the inhibition does not oscillate nearly as much with source position.

Also shown in Fig. 4 is the case of a 100-period Bragg region for $\kappa L = 2$ (dashed curves). The behavior is similar to that of the 10-period case except that the enhance-

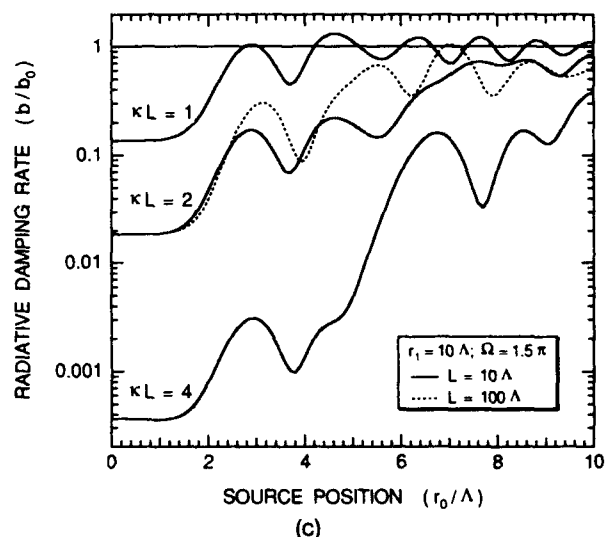
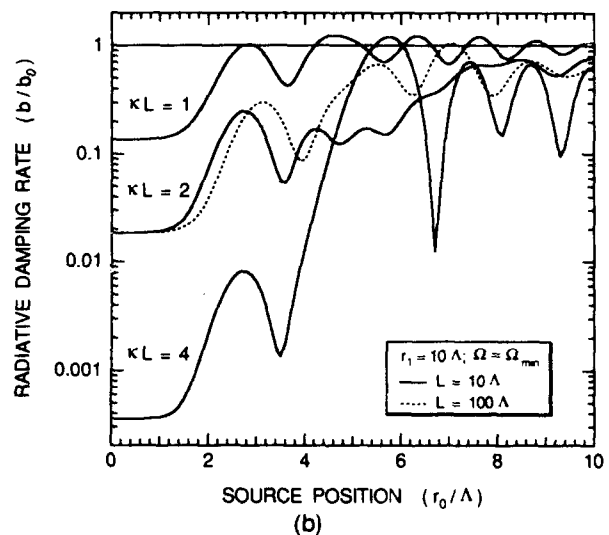
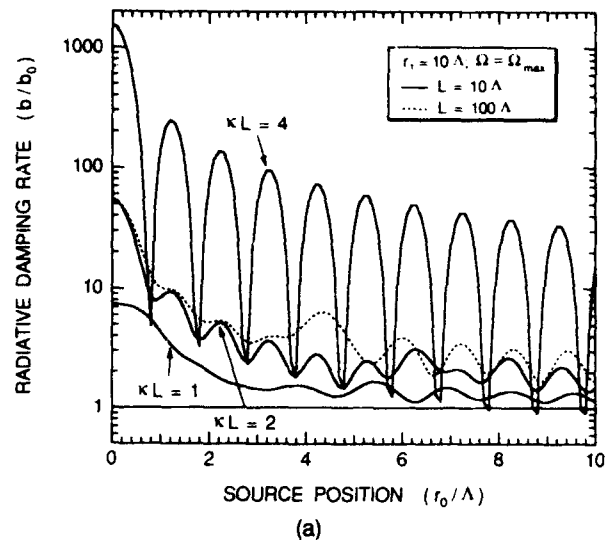


Fig. 4. Normalized radiative damping rate versus source position for various index modulation strengths: (a) grating phase Ω chosen to give maximum damping rate (maximized enhancement) in Fig. 3, (b) grating phase Ω chosen to give minimum damping rate (maximized inhibition) in Fig. 3, (c) grating phase $\Omega = 3\pi/2$.

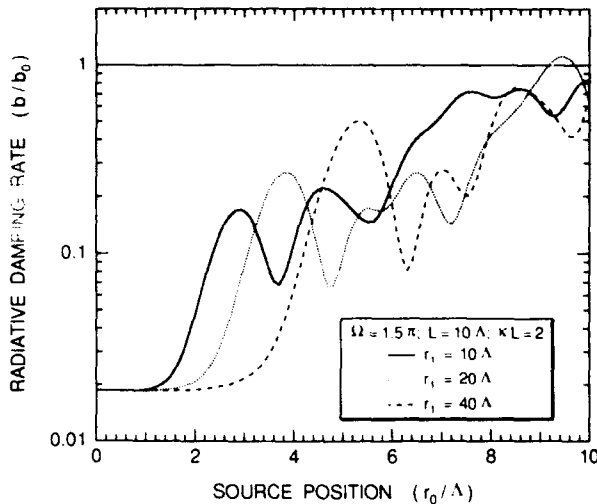


Fig. 5. Normalized radiative damping rate versus source position for various inner radii of a 10-period Bragg region.

ment and the inhibition remain large for a greater range of source positions than in the 10-period case. The reason for this slight difference can be explained as follows. The field emitted from a given source position can be well described by a finite number of azimuthal waves, as implied in Eqs. (13) and (14). Over the 10-period Bragg region the highest-order waves are still somewhat not cylindrically symmetric and hence are not strongly reflected by the structure. However, because the 100-period Bragg region covers a much larger radial extent, the highest-order waves can propagate further out radially and more closely approach cylindrically symmetric waves. Hence these waves are more strongly reflected in the 100-period case, even though the coupling-constant-grating-length product κL is the same in both cases.

This phenomenon is even more apparent in Fig. 5, where the inner radius of the Bragg structure (r_1) is varied for a fixed grating length. In Fig. 5 the radiative damping rate is plotted as a function of source position for three different inner radii of a 10-period Bragg structure. In this plot the grating phase is chosen to give maximum inhibition ($\Omega = 3\pi/2$). The farther from the center the Bragg structure is placed, the larger the radial extent of the region over which the source experiences maximum inhibition. Moving the inner radius of the Bragg structure from 10 to 40 periods from the center almost doubles the radius of the maximum inhibition region from 2 to 4 grating periods. The reason for this behavior was explained above: the farther from the center the Bragg reflection is occurring, the more of a chance the source waves have to approach their asymptotic, cylindrically symmetric form. Hence, for either enhancement or inhibition of the source radiation, it is desirable to push Bragg reflections as far from the center as possible, either by extending the inner radius of a strongly modulated, short grating or by using a more weakly modulated, long grating. In a practical structure the maximum radial extent of the Bragg cavity would be limited by propagation loss, which we do not consider here.

In Figs. 6 and 7 we turn to the issue of frequency shifts caused by the presence of the cylindrical Bragg cavity. A natural question to consider when using a Bragg cavity,

whose reflectivity properties are strongly dependent on frequency, is whether it is possible for the cavity to shift the frequency of the line source enough to push it outside the band gap of the cavity. If such a shift were possible, then the above numerical analysis, which assumes that the source is always tuned to the Bragg frequency, would be incorrect. The correct analysis, accounting explicitly for the interdependence of the lifetime and frequency variations, would be more complicated. Fortunately, as we show below, it is an excellent approximation to assume that the cavity-induced frequency shift is negligible as far as Bragg-reflectivity effects are concerned.

We first consider the question of how rapidly Bragg reflectivity effects vary with frequency detuning. Thus far in the analysis we have assumed that the source is exactly tuned to the Bragg frequency, or $\omega_s n = \omega_B$. This choice is made in the calculation of the field reflection coefficient from the Bragg structure with the coupled-wave equations (19). In Fig. 6 we plot the radiative damping

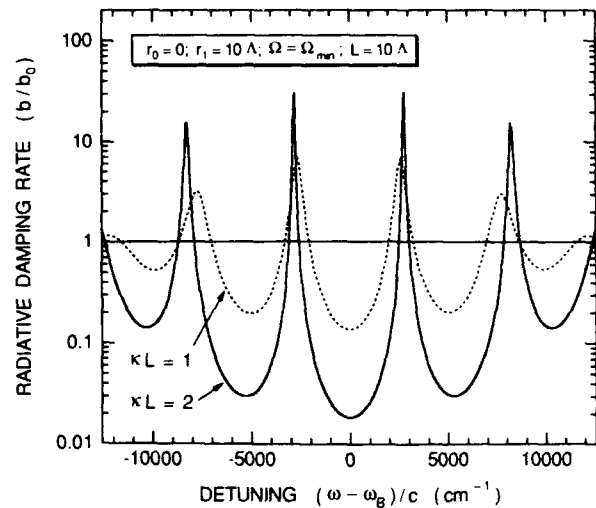


Fig. 6. Normalized radiative damping rate versus detuning of the source frequency from the Bragg frequency defined by the Bragg structure. The line source is placed at the center, and the grating phase is chosen to give inhibition at the Bragg frequency.

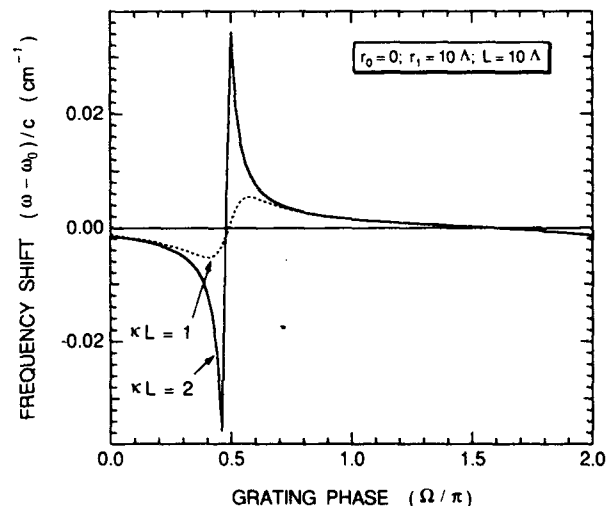


Fig. 7. Cavity-induced frequency shift of the line source versus grating phase for a full range of phases ($0 \rightarrow 2\pi$) and source placement at the center.

rate as a function of frequency detuning from the Bragg frequency. We assume a Bragg structure designed for a source emitting at $\lambda_0/n = 0.5 \mu\text{m}$, so that $\Lambda = 0.25 \mu\text{m}$, and ω_B is fixed; we vary the source frequency $\omega = \omega_0 n$. The source is placed at the center ($r_0 = 0$), and the grating phase Ω is chosen to give maximum inhibition at the Bragg frequency. The plot shows expected oscillations in the damping rate, with the maximum inhibition occurring at the Bragg frequency. Note that the cavity resonances that produce local maxima in inhibition are more closely spaced for the case of weaker modulation ($\kappa L = 1$), since the radial extent of the cavity is effectively larger. It is important to note the frequency scale in this plot: the radiative damping rate does not change significantly over a $>1000\text{-cm}^{-1}$ range of frequencies around the Bragg frequency.

In Fig. 7 we examine typical frequency shifts that are induced by the presence of the cavity. The plot is similar to Fig. 3, except that we show variation in frequency shift, $\Delta\omega/c$, instead of damping rate, as a function of grating phase Ω . The source is again placed exactly at the center, so that the plot gives a good indication of the full range of possible frequency shifts for any source position. The frequency shifts are generally tiny, except in the vicinity of the enhancement resonance around $\Omega = \pi/2$. Even at resonance the maximum frequency shift never exceeds $\pm 0.05 \text{ cm}^{-1}$, which is considerably less than the 1000-cm^{-1} range over which Bragg detuning can be considered negligible. Therefore the assumption that the source always remains tuned to the Bragg frequency is a good one.

4. CONCLUSIONS

It has been shown that it is indeed possible either to enhance or to inhibit the emission of radiation from a current line source located inside a cylindrically symmetric periodic structure. Whether the source experiences enhancement or inhibition depends on the phase of the periodic grating with respect to the center of the cylindrical system. When the line source is placed exactly at the center of the system, it tends to experience inhibition for most of the range of grating phases; only a very narrow phase range leads to enhancement. This result indicates that fabrication of a practical device that is designed to inhibit radiation will probably be easier than fabrication of an enhancement device because of tighter tolerances in the required grating phase.

Perhaps even more interesting is the result that significant coupling of source radiation to the cylindrical Bragg cavity occurs not only for placement of a line source at the center but also for a substantial region of source positions in the vicinity of the center. This result is important for practical reasons, since it suggests that spontaneous emission from sources located anywhere in an appreciable cross-sectional area of a cylindrically symmetric device can be enhanced or inhibited by cylindrical periodic structures of practical dimensions surrounding the area. Potential applications include planar waveguide devices as well as cylindrically symmetric lasers and amplifiers that operate along the cylinder axis.

It is found that one can increase the region of source placement for which significant radiative effects occur by

pushing the Bragg reflection from the periodic structure out to larger radial extents. This pushing can be accomplished either by an increase in the inner radius of the beginning of the periodicity or by use of a long (radially) grating with weak modulation, so that the reflection is spread out radially. The maximum radial extent of the Bragg cavity in a practical structure ought to be limited only by propagation losses.

Finally, an analysis of the harmonically oscillating current line source frequency shifts that are caused by the presence of the cylindrical Bragg cavity finds the shifts to be small. By comparing these shifts in the variation of Bragg reflectivity of the cavity with frequency, we find that the source remains exactly tuned to the Bragg frequency to a good approximation, even in cases of strong enhancement or inhibition of radiation.

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18. This statement is true only for a nonzero source position; for $r_0 = 0$, the source field is inherently singular at the center of the structure. However, this singularity does not affect the analysis, which is concerned only with the well-behaved reflected field at the source position.
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